On the Geometry of the Firewall

Partially mixed states in SYK

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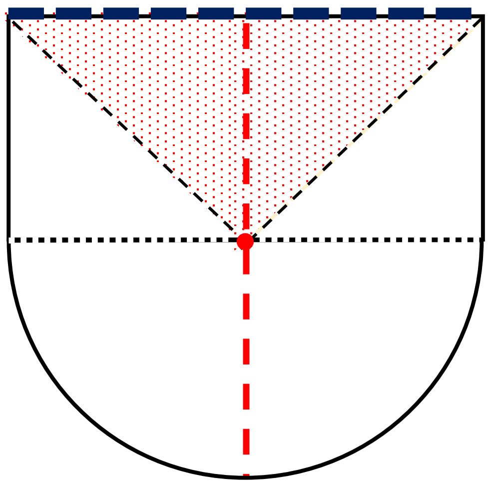
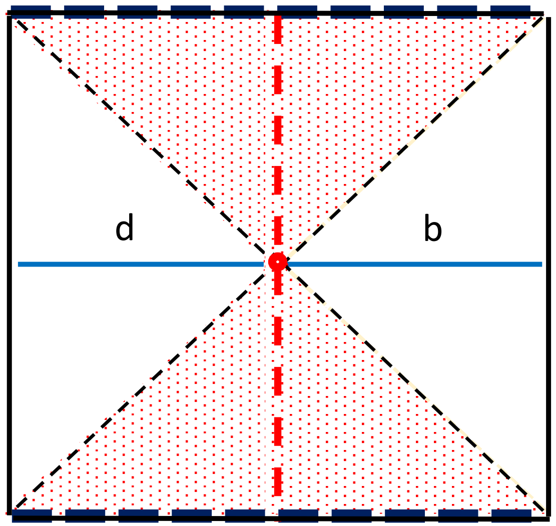
Holographic Quantum Matter
05/05/2018

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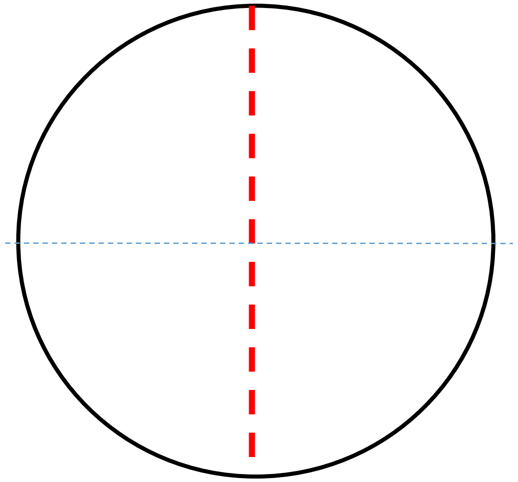

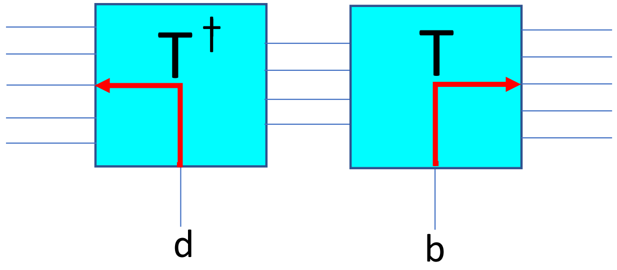
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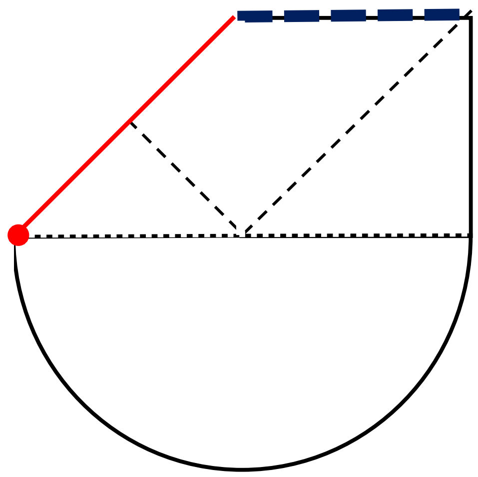

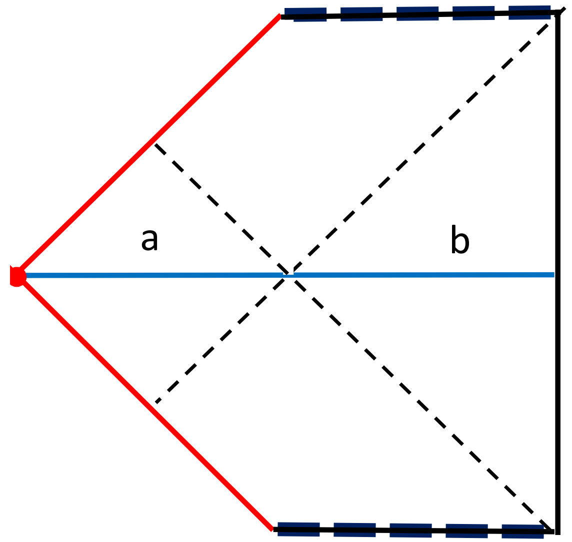
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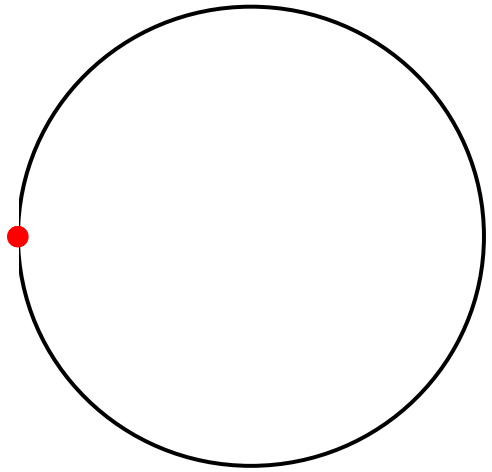

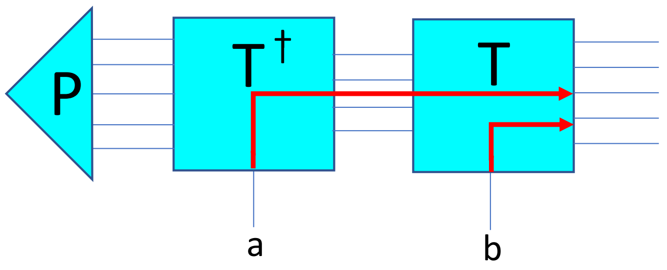
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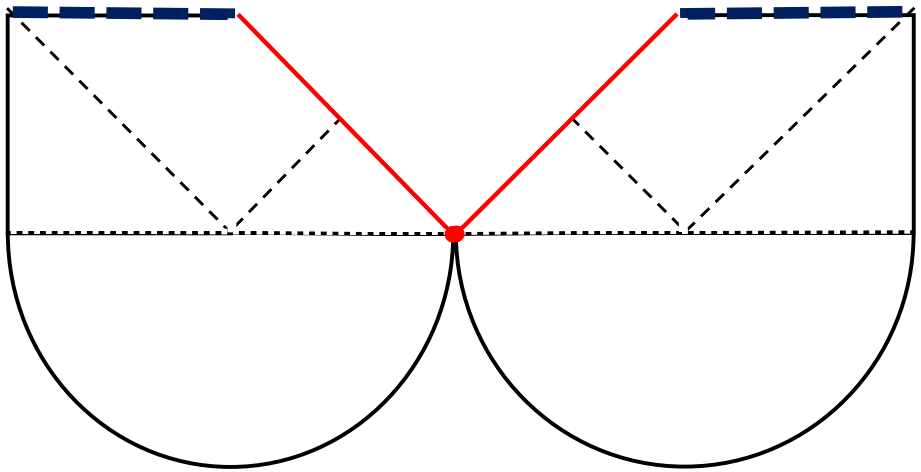

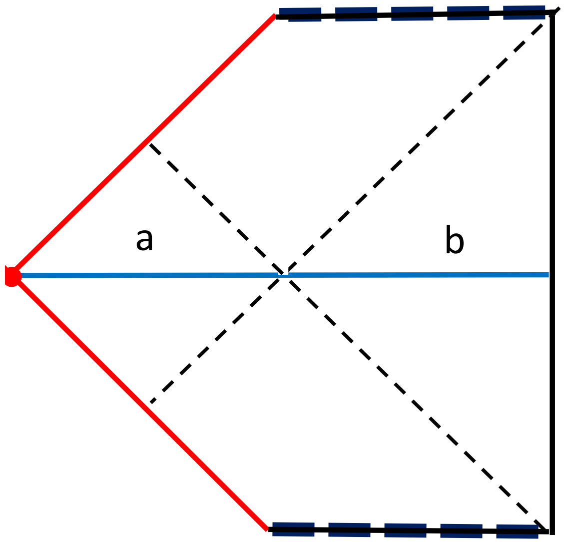
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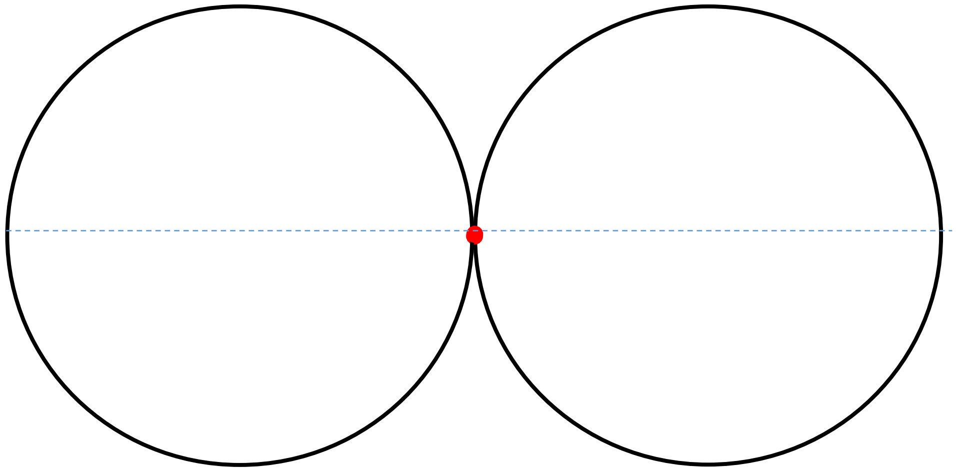

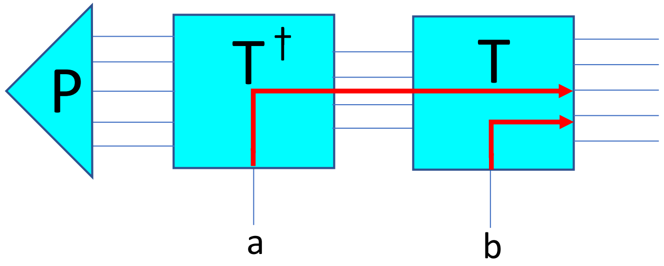
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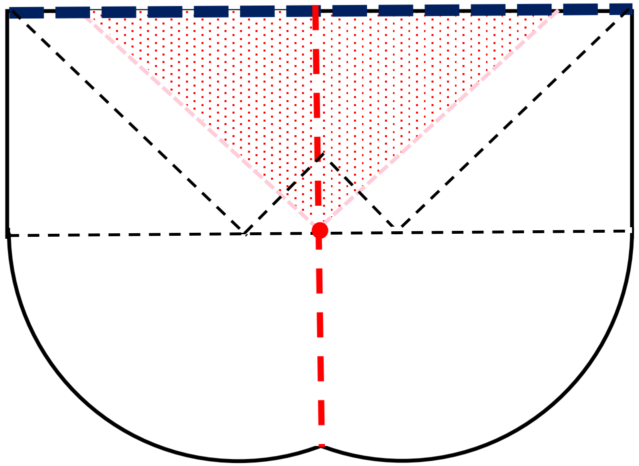

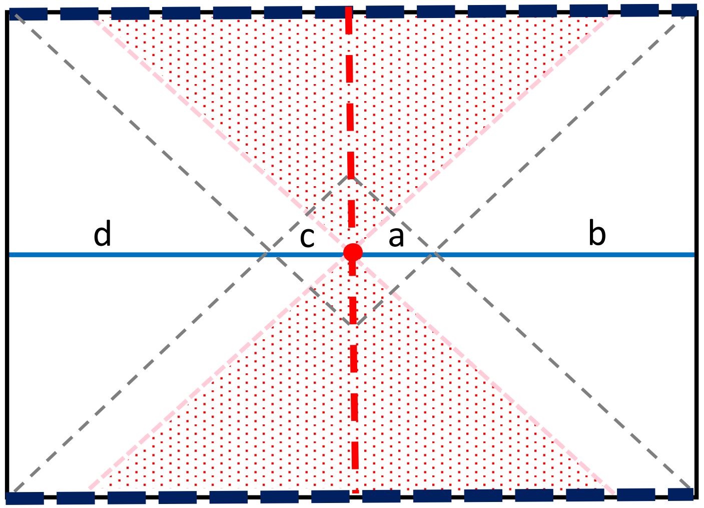
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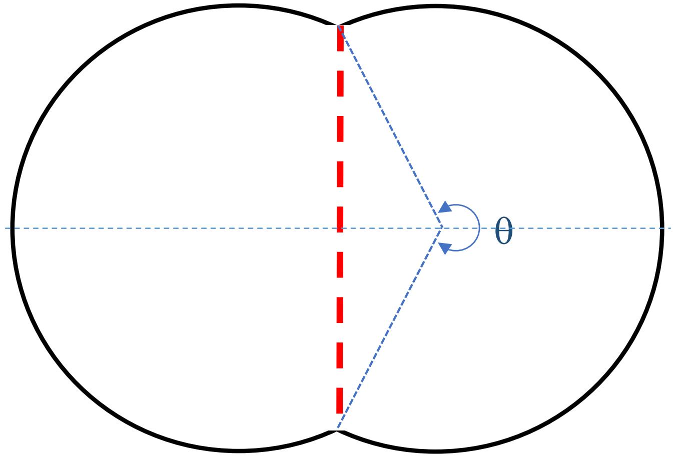
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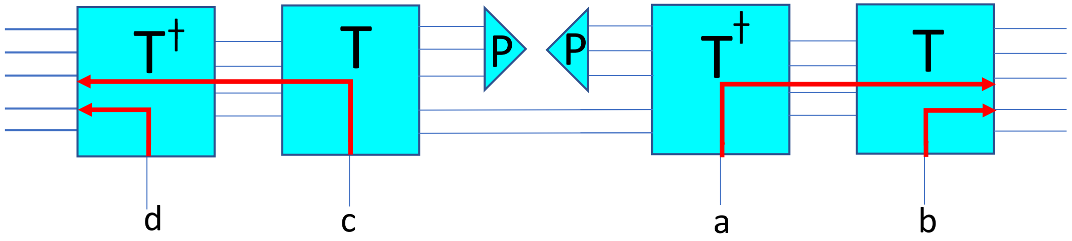
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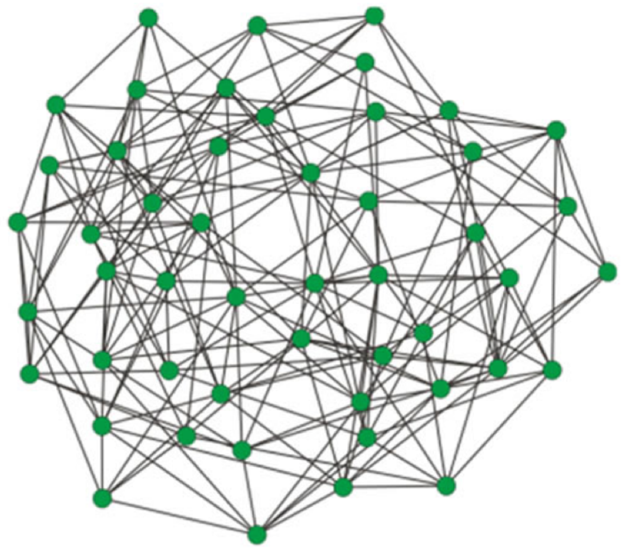
SYK model = 1D many body QM with maximal chaos

$$H = \sum_{ijk\ell} J_{ijk\ell}\psi^i\psi^j\psi^k\psi^\ell$$

random couplings

$$\{\psi^i, \psi^j\} = \delta^{ij}$$

N majorana variables



IR limit of SD equations

$$\int d\tau' G(\tau,\tau')\Sigma(\tau',\tau'') = -\delta(\tau-\tau'') \ , \qquad \Sigma(\tau,\tau') = J^2 \left[G(\tau,\tau')\right]^{q-1}$$

are invariant under 1D diffeomorphisms

$$G(\tau,\tau') \to \left[f'(\tau)f'(\tau')\right]^{\Delta} G(f(\tau),f(\tau')) \ , \quad \Sigma(\tau,\tau') \to \left[f'(\tau)f'(\tau')\right]^{\Delta(q-1)} \Sigma(f(\tau),f(\tau'))$$

→ IR effective theory is dominated by a dynamical

Goldstone mode = 1D reparametrizations f(τ)

$$S[f] = -C\int_0^\beta d\tau \left(\{f,\tau\} + \frac{2\pi^2}{\beta^2} f'^2\right) \qquad \{f,\tau\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

$$= -C\int_0^\beta d\tau \{F,\tau\}, \qquad F \equiv \tan\left(\frac{\pi f(\tau)}{\beta}\right) \qquad F \to \frac{aF+b}{cF+d}$$

Schwarzian QM = exactly solvable

➔

should be able to compute anything we want!

Low dimensional holography

SYK model $\leftrightarrow$ 2D dilaton gravity

$$S_{2D} = \int d^2x \sqrt{-g}\, \Phi(R+\Lambda) + S_{\text{matter}}$$

Almheiri, Polchinski; Jensen; Maldacena, Stanford, Yang; Engelsoy, Mertens, HV; Kitaev

equivalent to:

charged particle
on hyperbolic plane
w/ constant B-field

$$Z(\beta) = \int_{\mathcal{M}} \mathcal{D}f \, e^{-S[f]}$$

Partition function

$$\mathcal{M} = \mathrm{Diff}(S^1)/SL(2,\mathbb{R})$$

integral over energy E = k^2

with continuous spectral density

$$\rho(E) = \sinh\left(2\pi\sqrt{E - 1/4}\right)$$

Stanford, Witten

$$Z(\beta) = \int_0^\infty d\mu(k)\, e^{-\beta E(k)}, \qquad d\mu(k) = dk^2 \sinh(2\pi k).$$

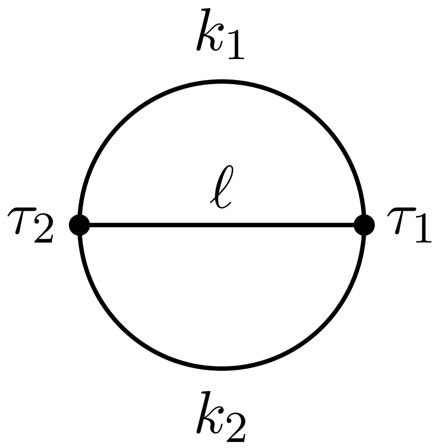
$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \frac{1}{Z}\int_{\mathcal{M}} \mathcal{D}f\, e^{-S[f]}\, \mathcal{O}_1 \dots \mathcal{O}_n$$

Correlation functions

$$\mathcal{O}_\ell(\tau_1,\tau_2) \equiv \left(\frac{\sqrt{f'(\tau_1) f'(\tau_2)}}{\frac{\beta}{\pi} \sin \frac{\pi}{\beta}[f(\tau_1) - f(\tau_2)]} \right)^{2\ell}$$

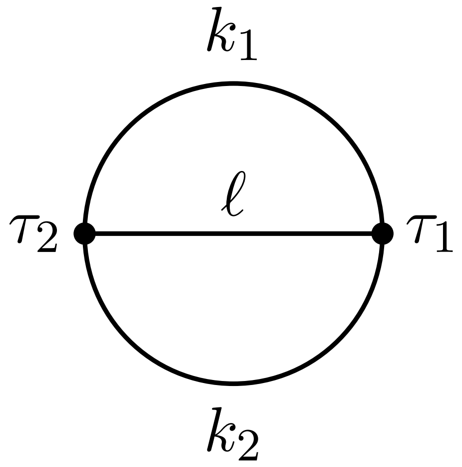
Two-point function

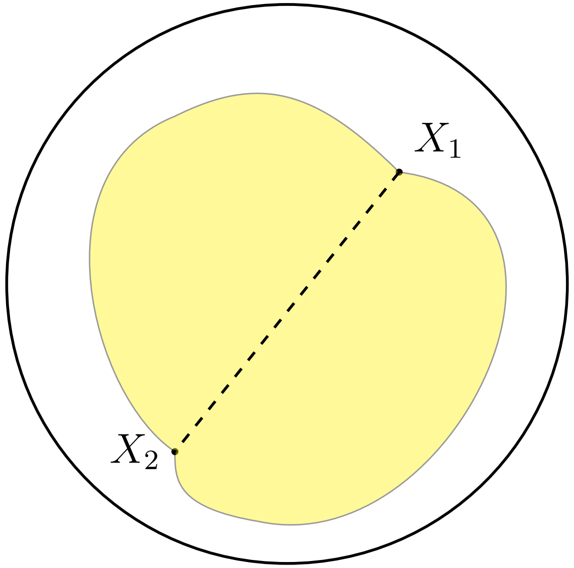
$$\langle \mathcal{O}_\ell(\tau_1,\tau_2) \rangle = \int \prod_{i=1}^{2} d\mu(k_i)\; \mathcal{A}_2(k_i,\ell,\tau_i).$$



Two point function

$$\mathcal{A}_2(k_i, \ell, \tau_i) \; = $$

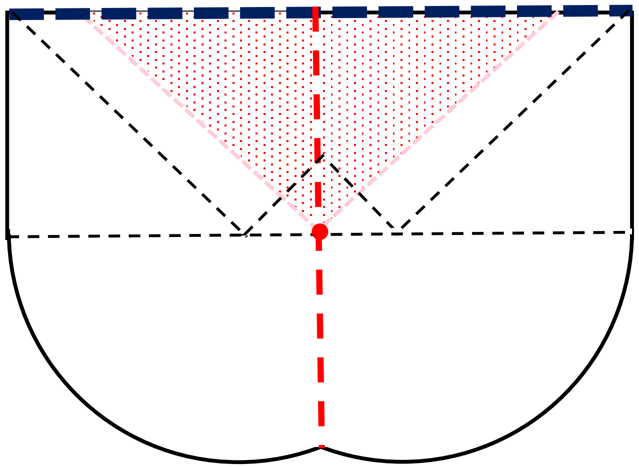


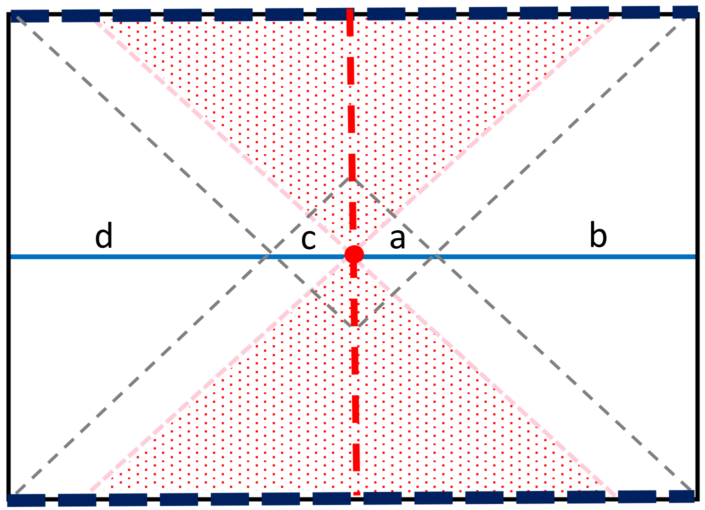


Kitaev
Gu, Lucas, Qi

$$\boxed{\mathcal{A}_2(k_i, \ell, \tau_i) \; = \; e^{-(\tau_2-\tau_1)k_1^2-(\beta-\tau_2+\tau_1)k_2^2} \; \frac{\Gamma(\ell \pm ik_1 \pm ik_2)}{\Gamma(2\ell)}}$$

Mertens, Turiaci, HV
c.f. Bagrets et al

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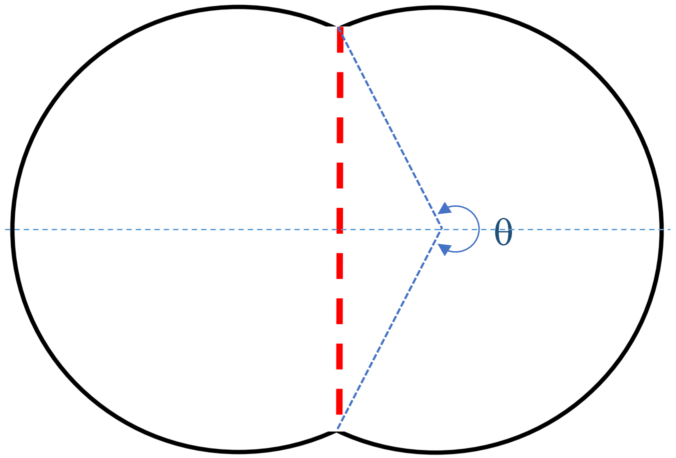
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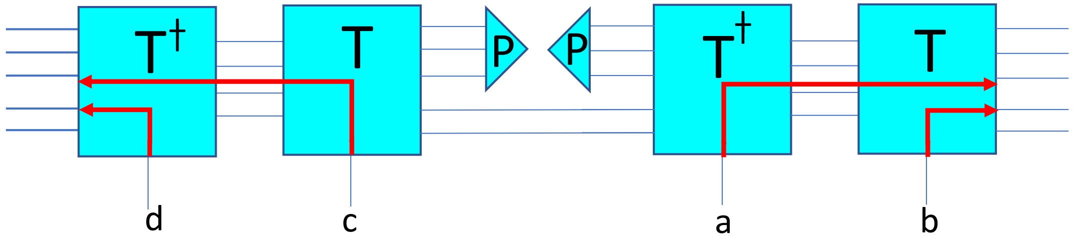
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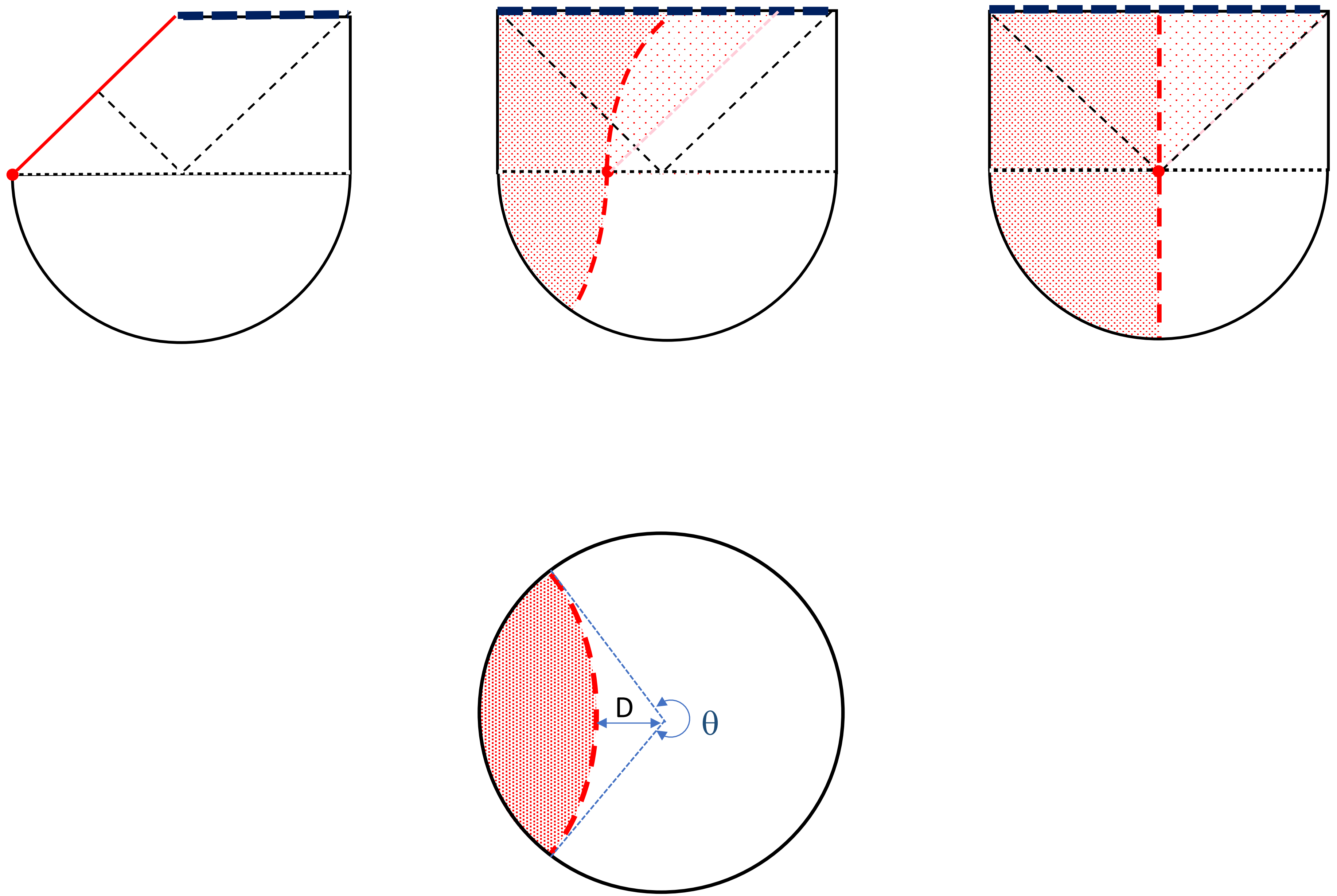
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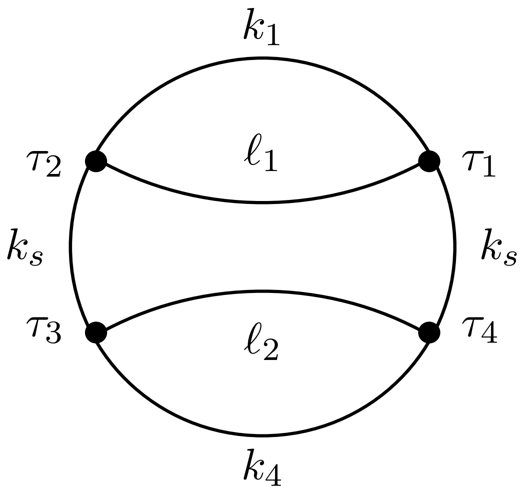
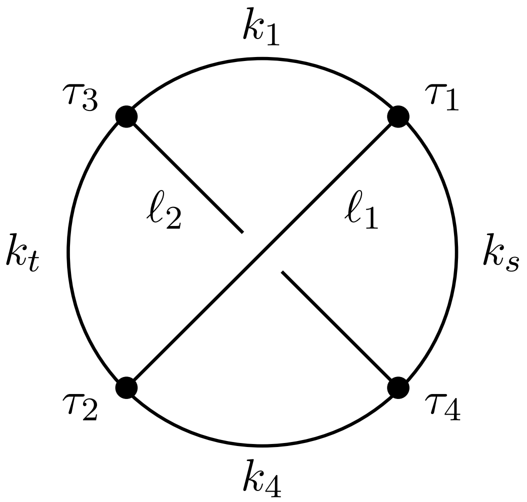
Four-point function

$$\left\langle \mathcal{O}_{\ell_1}(\tau_1,\tau_2)\,\mathcal{O}_{\ell_2}(\tau_3,\tau_4)\right\rangle \; = \; .$$

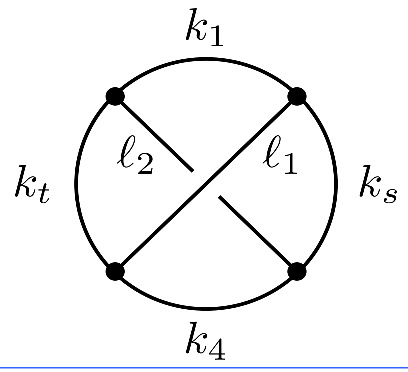


OTO four-point function

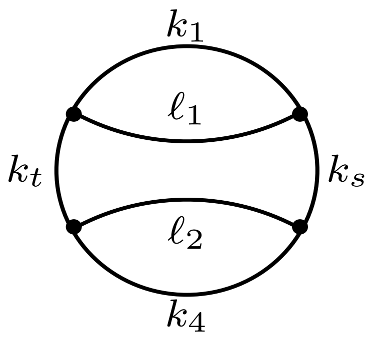
$$\left\langle \mathcal{O}_{\ell_1}(\tau_1,\tau_2)\,\mathcal{O}_{\ell_2}(\tau_3,\tau_4)\right\rangle_{\mathrm{OTO}} \; =$$



R-matrix



$$= \quad R_{k_s k_t}\left[\begin{smallmatrix} k_4 & \ell_2 \\ k_1 & \ell_1 \end{smallmatrix}\right]$$



The R-matrix of the Schwarzian is found to be equal to a classical 6j-symbol of SU(1,1)

$$R_{k_s k_t}\left[\begin{smallmatrix} k_4 & \ell_2 \\ k_1 & \ell_1 \end{smallmatrix}\right] = \left\{\begin{matrix} \ell_1 & k_4 & k_s \\ \ell_2 & k_1 & k_t \end{matrix}\right\} = \sqrt{\Gamma(\ell_1 \pm ik_2 \pm ik_s)\Gamma(\ell_3 \pm ik_2 \pm ik_t)\Gamma(\ell_1 \pm ik_4 \pm ik_t)\Gamma(\ell_3 \pm ik_4 \pm ik_s)}$$
$$\times\, \mathbb{W}(k_s, k_t; \ell_1 + ik_4, \ell_1 - ik_4, \ell_3 - ik_2, \ell_3 + ik_2),$$

$\mathbb{W}$ = Wilson function

Matches with the gravitational shockwave amplitude

Groenevelt